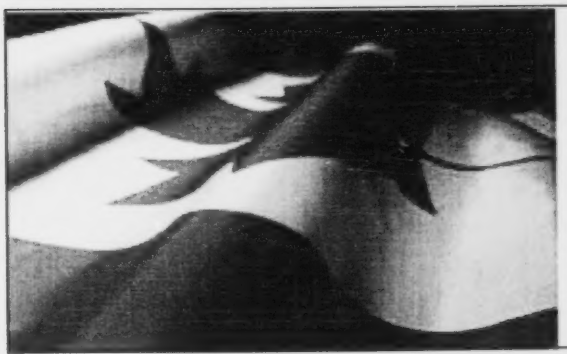

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A Survey of Taylor-Type Monetary Policy Rules

by

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Abstract

There are numerous ways to specify monetary-policy reaction functions in economic models. In recent years, the practice of using the Taylor rule, or more generally, Taylor-type rules, as a monetary policy rule has become more wide spread in the economic literature. The structure of this rule is simple and tractable but at the same time captures the essence of the behaviour of the monetary authority. In this paper, we present a brief survey of the literature on the properties of Taylor-type monetary policy rules. We review the modifications made to the original Taylor rule and their justifications, the empirical and theoretical issues related to the use and design of the rule, the use of calibrated versus estimated Taylor-type rules in economic models, and the practices of choosing an optimal monetary policy rule.

Sommaire

Il existe bien des façons de préciser les fonctions de réaction de la politique monétaire dans les modèles économiques. Ces dernières années, le recours à la règle de Taylor, ou de manière plus générale, aux règles tayloriennes, à titre de règle de la politique monétaire, est devenu de plus en plus répandu dans les ouvrages économiques. La structure de cette règle est simple et souple et saisit par ailleurs l'essence même du comportement de l'autorité monétaire. Dans le présent document, nous effectuons un bref survol des ouvrages traitant des propriétés de règles tayloriennes appliquées à la politique monétaire. Nous passons en revue les modifications apportées à la règle initiale de Taylor et leur justification, les enjeux empiriques et théoriques liés à la conception et à l'utilisation de la règle, l'utilisation de règles tayloriennes calibrées versus estimées dans les modèles économiques, et les pratiques de sélection d'une règle de politique monétaire optimale.

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Introduction

Modelling the reaction of the monetary authority to changes in economic conditions has long been an important task to model builders. This "reaction function" summarizes how the monetary authority utilizes its instrument – namely, the short-term interest rate – to stabilize inflation and output fluctuations in response to demand or supply shocks. The policy rule thus plays an important role in determining the dynamics of a macroeconomic model.

Numerous types of monetary policy rules have been discussed in the economic literature. The one that is more widely experimented with in macroeconomic models and analysed in the literature in recent years has been the monetary policy rule of Taylor (1993). The rule stipulates that the instrument of the monetary authority reacts to two key goal variables: deviations of contemporaneous inflation from a pre-set target rate and deviations of contemporaneous real output from its potential level. The structure of this rule is simple and tractable but at the same time captures the essence of the behaviour of the monetary authority.

Since the early inception of the original Taylor rule, there have been numerous modifications to it in order to take into account other elements of the working of monetary policy. The literature on the properties of the Taylor rule and other Taylor-type rules is immense. This paper presents a brief survey of this literature. Specifically, the paper focuses on the experience of other researchers in estimating and formulating Taylor-type rules and how such rules affect the dynamics of macroeconomic models.

1. The original Taylor rule

The introduction of rational expectations in economics in the mid-1970s has revolutionized the direction and thinking of macroeconomic research and policy recommendations. Based on solutions to time-inconsistency problems, one of the

recommendations of the literature is that policy rules are superior to discretion.[1] The rationale is that discretion results in either consistent but sub-optimal planning or in economic instability. Given these theoretical recommendations, the key issue is how to find a practical rule that is simple and easily understood such that it is obvious to economic agents when policymakers deviate from the policy.

The simple monetary rule proposed by Taylor (1993) is an attempt to address this issue. It is specified as:

$$i_t = r^* + \pi_t + \alpha(\pi_t - \pi^*) + \beta\tilde{y}_t \,, \tag{1.1}$$

where i is the nominal Federal funds rate; r^* is the equilibrium real Federal funds rate; π_t is the rate of inflation over the previous four quarters; π^* is the target inflation rate; and $\tilde{y}_t$ is the per cent deviation of real GDP from its target level. In this framework, the Federal funds rate rises if inflation rises above its target value π^* or real GDP rise above its target level. When both inflation and GDP are on target, the nominal Federal funds rate will be equal to the sum of the equilibrium real value r^* and target inflation π^*. For ease of exposition, the nominal interest rate rule can be rewritten as:

$$i_t = \delta + (1+\alpha)\pi_t + \beta\tilde{y}_t \,, \tag{1.2}$$

where $\delta = r^* - \alpha\pi^*$. Equivalently, the rule can be written in terms of the real interest rate $r_t = i_t - \pi_t$:

$$r_t = \delta + \alpha\pi_t + \beta\tilde{y}_t \,. \tag{1.3}$$

Taylor (1993) sets both the inflation target π^* and the equilibrium real rate r^* equal to 2 and assigns an equal weight of 0.5 to the inflation and output gap terms. Using these values, he shows that this simple rule tracks the actual path of the Federal funds rate reasonably well.

The parameter values in (1.3) reflect the preferences of the monetary authority, and a stability condition of α greater than zero is frequently assumed. It is easy to see that if α were less than zero, rising inflation would, *ceteris paribus*, cause the real

[1] See Kydland and Prescott (1977) and Calvo (1978).

interest rate to fall, providing a positive stimulus to output. This also implies that the estimated coefficient on inflation in a nominal interest rate rule of equation (1.2) must be greater than 1 for the stability condition to hold. Although Taylor proposes the parameter values of α and β (both are assumed to equal 0.5) as reasonable benchmarks, the optimal value of these parameters would ultimately depend on the structure of the model under consideration.

2. Empirical issues related to Taylor-type rules

Following Taylor (1993), a number of researchers have estimated the Taylor rule and variants of the rule (referred hereafter as Taylor-type rules). This area of research has uncovered a number of issues that must be taken into consideration when designing Taylor-type rules. First, given the fact that there are different ways to measure inflation and to estimate the equilibrium real interest rate and potential output, the robustness of estimation results can be sensitive to data selection. Second, the timing of information faced by the monetary policy authority can affect the structure of the policy rule and policy outcome. This causes some researchers to question the appropriateness of using contemporaneous observations of inflation and output in an instrument rule while in reality central banks must rely on lagged information. Others have questioned the merit of using revised data to estimate a historical policy rule since policy makers typically rely on real-time data when setting interest rate paths. Third, the structure of the Taylor rule assumes that policy makers consider only current information when making policy decisions. This view is at odds with the forward-looking nature of central banks. As such, a number of researchers have come to favour forward-looking versions of the rule. Finally, a widespread modification to the Taylor rule is to include terms representing interest rate smoothing behaviour by central banks. This section will look at each issue in more detail.

2.1 Measuring inflation

There is no consensus on which price variables should be used and how long the inflation period should be in Taylor-type rules. Taylor's original specification measured

inflation as the year-over-year change in the price deflator for GDP (Taylor 1993). Subsequently, other researchers have experimented with alternative price indexes. Kozicki (1999) compares the tractability of the Taylor rule with four alternative measures of inflation for the U.S. These measures include year-over-year inflation calculated with the CPI, core CPI, the GDP price deflator, and a measure of expected inflation obtained from an average of private-sector forecasts. Kozicki then uses a Taylor rule to calculate interest rate recommendations (that is, what Federal funds rate would have been had the Federal Reserve followed the Taylor rule) that would result from these alternative inflation measures. As expected, the recommendations differ significantly. Differences in interest rate settings range from a minimum of 0.6 percentage points to a maximum of 3.8 percentage points. Note that this exercise takes inflation and output as given and thus does not allow for feedbacks from the interest rate.

2.2 Measuring potential output

The inclusion of the output gap in the instrument rule has been justified on a number of grounds. A common view is that the output gap represents monetary policy makers' goal of output stability. Others view the output gap as a proxy for expected future inflation. Favero and Rovelli (1999) provide empirical evidence supporting this claim. Levin *et al.* (1999) use this argument to suggest that the parameter on the output gap should never fall too low (certainly greater than 0.6 or 0.8 based on their calculations).

Regardless of the rationale for the inclusion of the output gap, the subjective nature of measuring potential output causes uncertainty surrounding its accuracy. Taylor (1993) measures potential output by fitting a time trend to actual output. Other measures used in the literature include regressing actual output on segmented linear trends and quadratic trends, HP filters, and more structural approaches such as that developed for the U.S. by the Congressional Budget Office.[2] However, different potential output measures can result in quite different policy recommendations. The robustness analysis of Kozicki

[2] Woodford (2001) and McCallum and Nelson (1999) caution against the use time trends to estimate potential output on the grounds that potential output is subject to exogenous shocks not captured by these exogenous estimates.

(1999) shows that interest rate recommendations using alternative measures of potential output differ by a minimum of 0.9 percentage points to a maximum of 2.4 percentage points.

2.3 Equilibrium real interest rate

Perhaps the most difficult variable to estimate is the equilibrium real interest rate. The additive construction of Taylor-type rules means that the estimation of the equilibrium real interest rate has a direct impact on interest rate recommendations. Since a Taylor-type rule typically consists of an unobserved equilibrium real interest rate and in most cases an unobserved inflation target, a frequently used approach to deal with this problem is to assume a value for one of the two variables and then estimate the other. Examples of this approach are found in Kozicki (1999), Clarida, Galí and Gertler (2000) (thereafter as CGG), and Judd and Rudebusch (1998) who calculate the equilibrium real interest rate as the difference between the average Federal funds rate and the average inflation rate. Given this estimate of the equilibrium real interest rate, the estimated inflation target is then extracted from the constant term in the estimated Taylor-type rule. Alternatively, starting with an inflation target, the equilibrium real interest rate can be calculated from the constant term. More involved approaches include that of Rudebusch (2001) where the equilibrium real interest rate is estimated from an IS equation of

$$\tilde{y}_t = \vartheta_1 \tilde{y}_{t-1} + \vartheta_2 \tilde{y}_{t-2} - \theta\left(i_{t-1} - \pi_{t-1} - r^*\right) + v \,. \tag{2.1}$$

Rudebusch estimates that the equilibrium real interest rate r^* equals 2.2 per cent, similar to that used in the original Taylor rule. This estimate of the equilibrium real interest rate is then used in the constant term of a Taylor-type rule.

However, the assumption of a constant equilibrium real interest rate is itself questionable, making the measurement issue even more problematic. Kozicki (1999) shows that estimates of the equilibrium real interest rate for the U.S. vary with the sample period, suggesting that equilibrium real interest rates may not be constant. Rapach and Weber (2001) have also identified structural breaks in their measures of real interest rates for a number of countries, including Canada.

2.4 *Current versus lagged data*

The unavailability of current information to policy makers at the time decisions are made has led to a debate about whether to use current or lagged data in estimating interest-rate rules. In general, the empirical evidence does not show a substantial loss in performance when lagged data are used instead of current data (Levin *et al.* (1999), McCallum and Nelson (1999), Rudebusch and Svensson (1999)). The costs are small because inflation and output are persistent enough such that lags of the inflation rate and the output gap are good proxies for current values. Also, it is reasonable to claim that the central bank has more information about the state of the economy at the time interest rate changes are made than is captured by inflation and output alone (Batini and Haldane (1999)). It has been argued on this premise that using contemporaneous data instead of lagged data can be thought to implicitly include information that is not reflected in inflation and output measures (Kozicki (1999), Rudebusch and Svensson (1999)).

2.5 *Forward-looking rules*

Monetary authorities generally make policy decisions based on economic conditions expected in the future. Accordingly, a number of researchers prefer forward-looking, or forecast-based interest rate rules, to contemporaneous rules. The same arguments that are used to support contemporaneous rules over lagged rules extend to the debate over the appropriateness of forward-looking rules. It is not clear that forward-looking estimates have any advantage over contemporaneous or backward-looking versions of the rule owing to the persistent nature of inflation and output. Section 4.5 discusses this issue in more detail.

2.6 *Current-time versus real-time data*

It is well known that economic data go through numerous revisions after their original release, some of these revisions occurring even years later. Calculating the recommended interest rate response with the original Taylor rule using real-time data yields considerably different results than with current-time data.

Taylor (1999b) uses current-time data to demonstrate that if the Federal Reserve had set the Federal funds rate as recommended by the Taylor rule, the inflationary episode of the 1970s might have been avoided. However, Orphanides (2001) counters this argument by calculating the recommendations of Taylor's rule using the data available to decision makers at the time and shows that policy makers following the original Taylor rule would not have been able to avoid the inflationary episode.

2.7 Interest rate smoothing

Interest rate paths set by central banks tend to be fairly smooth, moving slowly in the same direction rather than frequently reversing the direction of change. Taylor-type rules are commonly modified to incorporate interest rate smoothing by including a lagged interest rate term. Sack and Wieland (1999) question whether interest rate smoothing is deliberate or simply the result of monetary policy reacting to persistent macroeconomic conditions. If interest rate smoothing reflects the reaction of monetary authorities to persistent macroeconomic variables, one would expect the coefficient on the lagged interest rate to be small or insignificant. However, estimated Taylor-type rules that include a lagged interest rate typically yield large and significant coefficients, indicating that interest rate smoothing is deliberate.

Several arguments have been put forth in favour of interest rate smoothing. Among these are explanations of why central banks would want to avoid frequent reversals in the direction of interest rate movements. Williams (1999) suggests that frequent reversals may appear as mistakes to the public, thus maintaining momentum in interest rate movements will maintain confidence in the central bank. Interest rate smoothing may also be a response to the unavailability of accurate economic information and the uncertainty associated with the monetary transmission mechanism. In light of these uncertainties regarding the true state of the economy, central banks may prefer to adjust slowly to their economic targets (Sack and Wieland (1999)).

3. Estimated Taylor-type rules

The literature on the empirical estimation of Taylor-type rules is extensive. Predominant in this literature are attempts to quantify the shift in macroeconomic policy towards inflation control experienced in many countries in the 1980s and early 1990s. However, empirical results can differ substantially between studies depending on the type of data or model used. This section will focus on studies by Taylor (1999b) (using the original Taylor rule), Judd and Rudebusch (1998) (incorporating interest rate smoothing in a modified Taylor rule), and CGG (2000) (incorporating forward-looking elements in a modified Taylor rule) in order to highlight the relative importance of the issues discussed above in affecting the empirical results. Note that the three studies surveyed in this section are for the U.S. only. There is very limited direct empirical estimation of Canadian Taylor-type rules in the literature.

3.1 Model specifications

In terms of model set up, Taylor (1999b) estimates the generalized nominal interest rate rule of:

$$i_t = \delta + (1+\alpha)\pi_t + \beta\tilde{y}_t \quad . \tag{3.1}$$

Judd and Rudebusch (1998) modify Taylor's rule by using an error-correction approach to allow for gradual adjustment in the Federal funds rate. Letting i_t^* be the recommended Federal funds rate that will be achieved through gradual adjustment, the modified Taylor rule is:

$$i_t^* = r^* + \pi_t + \alpha(\pi_t - \pi^*) + \beta_1\tilde{y}_t + \beta_2\tilde{y}_{t-1} \tag{3.2}$$

where the adjustment process is given by

$$\Delta i_t = \gamma(i_t^* - i_{t-1}) + \rho\Delta i_{t-1} \quad . \tag{3.3}$$

The coefficient γ measures the correction to the "error" in the interest rate setting while the coefficient ρ measures the "momentum" from last period's interest rate change. Combining these two equations gives the Taylor-type rule with interest rates smoothing to be estimated:

$$\Delta i_t = \gamma\xi - \gamma i_{t-1} + \gamma(1+\alpha)\pi_t + \gamma\beta_1\tilde{y}_t + \gamma\beta_2\tilde{y}_{t-1} + \rho\Delta i_{t-1} \tag{3.4}$$

where $\xi = r^* - \alpha\pi^*$.

CGG (1998, 2000) employ a partial-adjustment form of the Taylor rule that has been frequently used by other researchers[3] but further modify it to reflect forward-looking behaviour of monetary authorities. Their interest rate rule is:

$$i_t^* = i^* + \varphi\left(E\left[\pi_{t,k}\middle|\Omega_t\right] - \pi^*\right) + \beta E\left[\tilde{y}_{t,q}\middle|\Omega_t\right] \tag{3.5}$$

where i_t^* is the target nominal interest rate; i^* is the desired nominal interest rate when inflation and output are at their target values; $\pi_{t,k}$ is the change in the price between periods t and $t+k$; $\tilde{y}_{t,q}$ is a measure of the average output gap between periods t and $t+q$; Ω_t is the information set available at the time the interest rate is set. Interest rate smoothing is imposed via a partial-adjustment process of:

$$i_t = \rho(L)i_{t-1} + (1-\rho)i_t^* \quad ; \ \rho(L) = \rho_1 + \rho_2 L + ... + \rho_n L^{n-1} ; \ \ \rho \equiv \rho(1) \tag{3.6}$$

where i_t is the current interest rate set by the monetary authority. Combining (3.5) and (3.6) yields the instrument rule to be estimated as:

$$i_t = (1-\rho)\left[r^* - (\varphi-1)\pi^* + \varphi\pi_{t,k} + \beta\tilde{y}_{t,q}\right] + \rho(L)i_{t-1} + \varepsilon_t \tag{3.7}$$

where $r^* = i^* - \pi^*$ is the long-run equilibrium real rate and

$$\varepsilon_t = -(1-\rho)\left[\varphi\left(\pi_{t,k} - E\left[\pi_{t,k}\middle|\Omega_t\right]\right) + \beta\left(\tilde{y}_{t,q} - E\left[\tilde{y}_{t,q}\middle|\Omega_t\right]\right)\right]. \tag{3.8}$$

3.2 Variable definitions

Even though the estimated forms of the instrument rule are similar, these studies use a wide range of variables and methods to construct inflation rates and potential output. As suggested earlier, differences in measurement can lead to very different empirical results.

Taylor (1999b) defines inflation as the four-quarter average of the quarter-over-quarter change in the GDP price deflator and uses a Hodrick-Prescott filter of real GDP to measure potential output. The instrument rule is estimated over three sample periods:

[3] McCallum (2001) and Kozicki (1999)

the international gold standard era (1879Q1 to 1914Q4), the Bretton Woods era (1960Q1 to 1979Q4), and the post-Bretton Woods era (1987Q1 to 1997Q3).

Contrary to Taylor (1999b), Judd and Rudebusch (1998) use a variety of prices (the GDP price deflator, core CPI, and the personal consumption expenditure deflator) to measure the inflation rate. They also define inflation as the four-quarter average of the quarter-over-quarter change in the price level. For potential output, besides using the Congressional Budget Office definition of potential output, they also use other measures by fitting a segmented linear trend and a quadratic trend to actual output. Data for the U.S. economy is divided into three samples corresponding to Federal Reserve chairmanship: the Burns period (1970Q3 to 1978Q2), the Volcker period (1979Q3 to 1987Q2), and the Greenspan period (1987Q3 to 1997Q4).

For their baseline estimates CGG (2000) use the annualized quarterly growth rate of the GDP deflator to measure inflation and the CBO measure of potential output. Since their rule incorporates forward-looking elements, they use future actual values of inflation to proxy expected inflation. They also experiment with other measures of potential output and inflation (the CPI index for inflation, a quadratic trend of output in place of CBO potential output, and the gap between actual unemployment and trend unemployment in place of the output gap). The equilibrium real interest rate is the observed sample average of the real interest rate. Given the equilibrium real interest rate, the target inflation rate is then estimated jointly with the remaining parameters in the instrument rule. Sample periods are divided based on Federal Reserve chairmanship: the pre-Volcker period (1960Q1 to 1979Q2) and the Volcker-Greenspan period (1979Q3 to 1996Q4).

3.3 Estimation results

Both Taylor (1999b) and Judd and Rudebusch (1998) use OLS or non-linear least squares to estimate their interest rate rules. CGG, on the other hand, estimate using the generalized method of moments (GMM). Since the error term in equation (3.8) is a linear combination of forecast errors, it is orthogonal to the information set Ω_t.

Therefore, a set of instruments can be extracted from the information set for use in the GMM estimation. As instruments CGG use lags of the Federal funds rate, inflation, the output gap, commodity price inflation, and the spread between the long-term bond rate and the three-month Treasury Bill rate. The nominal Federal funds rate is used as the instrument of the monetary authority in these studies.[4]

Taylor (1999b)

Estimation results of Taylor (1999b) are summarized in Table 1.[5] Interpreting the results is straightforward. Since the estimated value of κ (note that $\kappa = 1+\alpha$ in equation (3.1)) for the 1960Q1 to 1979Q4 period is less than 1, this implies the coefficient on inflation α in equation (3.1) is negative, representing accommodative monetary policy. The estimated value of κ, however, is greater than one in the post 1987Q1 period, indicating a period of inflation stabilization policy by the Federal Reserve. These results are consistent with the high inflation episode of the 1970s and the subsequent policies of inflation control from the 1980s onward. The last sample period is also a time of increased concern over output stabilization, which is captured by the increase in the value of the coefficient on the output gap.

Table 1
Estimation results of Taylor (1999b)
(numbers in parentheses are t-statistics)

$i_t = \delta + \kappa\pi_t + \beta\tilde{y}$ (or $i_t = \delta + (1+\alpha)\pi_t + \beta\tilde{y}$ as in equation (3.1))				
Estimation period	δ	κ	Implied α	β
1960Q1 to 1979Q4	2.045 (6.34)	0.813 (12.9)	-0.187	0.252 (4.93)
1987Q1 to 1997Q4	1.174 (2.35)	1.533 (9.71)	0.533	0.765 (8.22)

[4] Although these studies are restricted to using U.S. data, there are studies that look at monetary policy rules of other countries. For example, Nelson (2000) estimates Taylor-type rules for seven separate policy regimes for the UK. Although the sample size in each period is small, to some extent he is able to obtain parameter estimates that can be explained by the monetary experience of the time. CGG (1998) also estimate Taylor-type rules for Germany, Japan, UK, France, and Italy. They are able to characterize monetary policy regimes in these countries in a similar manner.

[5] Results for the Bretton Woods era are not discussed here.

Judd and Rudebusch (2000)

The estimation results of Judd and Rudebusch (2000) exemplify the problems of estimating Taylor-type rules where estimated results are sensitive to how inflation and potential output are measured. For ease of comparison, the interest rate rules of Judd and Rudebusch (2000) are rearranged to isolate the nominal interest rate on the left hand side of equation (3.4):

$$i_t = \gamma[\zeta + (1+\alpha)\pi_t + \beta_1 \tilde{y}_t + \beta_2 \tilde{y}_{t-1}] + (1-\gamma) i_{t-1} + \rho \Delta i_{t-1} \quad . \tag{3.9}$$

Tables 2a to 2c report the estimation results of Judd and Rudebusch when different inflation and potential output definitions are used over different Federal Reserve chairmanship. The magnitude of interest rate settings can vary substantially depending on the particular definition of inflation and potential output used. In general, the variation is less across different definitions of inflation than potential output. Some of the variables are not significant and are dropped by Judd and Rudebusch in their final estimation.

One interesting finding is that the estimated coefficient on inflation, α, is statistically insignificant during the Burns period (see Table 2a), suggesting the nominal interest rate was not adjusting to inflation. The results also show that during the Burns period, the estimated coefficient on the "error correction" term γ is always larger than the estimated coefficient measuring interest rate momentum ρ. This implies that interest rate changes are driven more by an adjustment to the desired level and less by maintaining the momentum in interest rate movements. This relationship is reversed during the Greenspan period.

Table 2a
Estimation results of Judd and Rudebusch (1998)
The Burns period: 1970Q3 to 1978Q2
(numbers in parentheses are t-statistics)

$i_t = \gamma[\zeta + (1+\alpha)\pi_t + \beta_1 \tilde{y}_t + \beta_2 \tilde{y}_{t-1}] + (1-\gamma) i_{t-1} + \rho \Delta i_{t-1}$						
Definition of inflation and potential output	γ	ξ	α	β_1	β_2	ρ
GDP deflator, CBO potential output	0.58 (4.78)	0.71 (2.68)	-	-	0.89 (5.85)	0.26 (1.76)
PCE deflator, CBO potential output	0.51 (4.33)	0.95 (2.98)	-	-	0.86 (4.67)	0.17 (1.14)
Core CPI, CBO potential output	0.56 (3.46)	1.30 (4.17)	-	-	1.14 (6.12)	0.34 (1.99)
GDP deflator, segmented linear trend	0.52 (4.15)	1.54 (3.76)	-	-	1.07 (4.83)	0.25 (1.59)
GDP deflator, quadratic trend	0.59 (4.78)	-0.15 (-0.58)	-	-	0.88 (5.90)	0.26 (1.78)

When estimating the monetary policy rule over the Volcker period, Judd and Rudebusch note that the monetary authority is more responsive to the change in, rather than the level of, the output gap.[6] Therefore, the coefficients on the current and lagged output gap are restricted to sum to zero in their estimation (see Table 2b). Judd and Rudebusch obtain positive estimates of α, indicating a move towards non-accommodative monetary policy over this period.

Table 2b
Estimation results of Judd and Rudebusch (1998)
The Volcker period: 1979Q3 to 1987Q2
(numbers in parentheses are t-statistics)

$i_t = \gamma[\zeta + (1+\alpha)\pi_t + \beta_1 \tilde{y}_t + \beta_2 \tilde{y}_{t-1}] + (1-\gamma) i_{t-1} + \rho \Delta i_{t-1}$						
Definition of inflation and potential output	γ	ξ	α	β_1	β_2	ρ
GDP deflator, CBO potential output	0.44 (3.64)	2.42 (1.56)	0.46 (1.79)	1.53 (1.92)	-1.53 (-1.92)	-
PCE deflator, CBO potential output	0.29 (2.59)	1.46 (0.52)	0.54 (1.21)	2.58 (1.65)	-2.58 (-1.65)	-
Core CPI, CBO potential output	0.35 (3.03)	1.32 (0.57)	0.35 (1.11)	2.57 (2.07)	-2.57 (-2.07)	-
GDP deflator, segmented linear trend	0.43 (3.46)	2.29 (1.39)	0.48 (1.75)	1.57 (1.84)	-1.57 (-1.84)	-
GDP deflator, quadratic trend	0.43 (3.46)	2.23 (1.33)	0.50 (1.78)	1.60 (1.84)	-1.60 (-1.84)	-

[6] However, the authors did not provide evidence to support this claim.

The estimation results of Judd and Rudebusch during the Greenspan period (see Table 2c) are less robust across different definitions of inflation and potential output than in the Burns and Volcker periods. As the price and potential output measures vary, estimates of the response of monetary authority to the output gap (β) vary moderately but the response to inflation (α) varies dramatically.

Table 2c
Estimation results of Judd and Rudebusch (1998)
The Greenspan period: 1987Q3 to 1997Q4
(numbers in parentheses are t-statistics)

$i_t = \gamma[\zeta + (1+\alpha)\pi_t + \beta_1 \tilde{y}_t + \beta_2 \tilde{y}_{t-1}] + (1-\gamma) i_{t-1} + \rho \Delta i_{t-1}$						
Definition of inflation and potential output	γ	ξ	α	β_1	β_2	ρ
GDP deflator, CBO potential output	0.28 (5.27)	1.31 (2.26)	0.54 (2.96)	0.99 (7.46)	-	0.42 (4.00)
PCE deflator, CBO potential output	0.23 (4.30)	2.39 (3.56)	0.07 (0.37)	1.02 (5.66)	-	0.44 (3.89)
Core CPI, CBO potential output	0.25 (4.56)	1.00 (1.28)	0.37 (1.79)	1.15 (7.20)	-	0.49 (4.47)
GDP deflator, segmented linear trend	0.18 (3.56)	5.35 (3.52)	-0.99 (-1.93)	0.90 (3.99)	-	0.54 (4.88)
GDP deflator, quadratic trend	0.28 (4.77)	1.09 (1.80)	0.37 (1.90)	0.82 (7.05)	-	0.52 (4.94)

The gradual adjustment approach taken by Judd and Rudebusch results in actual interest rate responses that are smaller in magnitude than the Taylor rule of equation (3.1). It is easy to see that the estimated coefficient on the interest rate response to inflation ($1+\alpha$) is reduced by the error correction term γ, implying the ultimate response of nominal interest rate will be less than one.

CGG (2000)

To estimate their policy rule, CGG need estimates of the equilibrium real interest rate r^* and the target inflation rate π^*. CGG use the sample average of the observed real interest rate as a proxy for r^*, but allowing it to vary between sub-samples. Given r^*, the target inflation rate π^* is estimated jointly with the other parameters in equation (3.7). Tables 3a and 3b report the estimation results. Unless otherwise specified, these

results correspond to one-quarter-ahead expectations of the inflation and output gaps (that is, k = q = 1 in equation (3.7)).

To check for robustness, CGG use both GDP and CPI deflators to calculate inflation, and a quadratic trend and CBO's potential output to estimate the output gap. They also tried replacing the output gap by the deviation of the unemployment rate from a time trend. Other rules experimented with include a backward-looking version (where k = q = -1), and one where the expectation of the inflation gap is one year ahead (k=4) and that for the output gap is one quarter ahead (q=1). Note that the CGG rule of equation (3.5) is not directly comparable to equation (3.1) because the real interest rate is not explicitly included in the rule.[7] Overall, the results are qualitatively similar to the one-quarter-ahead horizon, suggesting that estimated rules might be fairly robust to alternative time horizons.

Table 3a
Estimation results of CGG (2000)
The pre-Volcker period: 1960Q1 to 1979Q2
(numbers in parentheses are t-statistics)

$i_t = (1-\rho)\left[r^* - (\phi-1)\pi^* + \phi\pi_{t,k} + \beta\tilde{y}_{t,q}\right] + \rho(L)i_{t-1} + \varepsilon_t$				
Definition of inflation and potential output	ϕ	β	π^*	ρ
GDP deflator, CBO potential output	0.83 (11.86)	0.27 (3.38)	4.24 (3.89)	0.68 (13.6)
GDP deflator, CBO potential output, k = q = -1	0.86 (12.28)	0.39 (4.88)	5.95 (3.1)	0.68 (13.6)
GDP deflator, CBO potential output, k = 4, q = 1	0.86 (17.2)	0.34 (4.25)	3.58 (2.52)	0.73 (18.25)
CPI deflator, CBO potential output	0.68 (11.33)	0.28 (3.50)	4.56 (8.60)	0.65 (13.0)
GDP deflator, quadratic trend output	0.75 (10.71)	0.29 (3.62)	4.17 (6.13)	0.67 (13.40)
GDP deflator, trend unemployment rate	0.84 (16.80)	0.60 (5.45)	3.80 (4.37)	0.63 (15.75)

[7] ϕ is qualitatively equivalent to $(1+\alpha)$ in equation (3.1).

Table 3b
Estimation results of CGG (2000)
The Volcker-Greenspan period: 1979Q3 to 1996Q4
(Numbers in parentheses are t-statistics)

$$i_t = (1-\rho)\left[r^* - (\phi-1)\pi^* + \phi\pi_{t,k} + \beta\tilde{y}_{t,q}\right] + \rho(L)i_{t-1} + \varepsilon_t$$

Definition of inflation and potential output	ϕ	β	π^*	ρ
GDP deflator, CBO potential output	2.15 (5.37)	0.93 (2.21)	3.58 (7.16)	0.79 (19.75)
GDP deflator, CBO potential output, k = q = -1	1.72 (6.14)	0.34 (1.78)	4.08 (7.28)	0.71 (14.2)
DP deflator, CBO potential output, k = 4, q = 1	2.62 (8.45)	0.83 (2.96)	3.25 (14.13)	0.78 (26.0)
CPI deflator, CBO potential output	2.14 (4.12)	1.49 (1.71)	3.47 (4.39)	0.88 (29.33)
GDP deflator, quadratic trend output	1.97 (6.16)	0.55 (1.83)	4.52 (7.79)	0.76 (15.20)
GDP deflator, trend unemployment rate	2.01 (7.08)	0.56 (1.37)	4.42 (10.05)	0.73 (14.60)

The empirical results of CGG (2000) are qualitatively similar to those of Taylor (1999b). In both cases the estimated coefficients on inflation and the output gap are larger during the Volcker-Greenspan period (from 1979Q3 to 1996Q4) relative to the pre-Volcker period (from 1960Q1 to 1979Q2). The estimated coefficient of the inflation rate ϕ is less than one during the pre-Volcker period but greater than one during the Volcker-Greenspan period. However, unlike the Taylor and Judd and Rudebusch results, the estimated coefficient on the output gap becomes insignificant during the Volcker-Greenspan period. Similar to the Judd and Rudebusch results, the magnitude of smoothing increases in the latter period. Cao *et al.* (2000) estimate a rule similar to CGG (2000) for the Canadian economy over the period 1988 to 1998. They found evidence of interest rate smoothing (with a coefficient of 0.8) in Canada. However, contrary to CGG's results, their estimated coefficient on the output gap is significant and close to unity.

CGG's estimated results are also sensitive to the choice of prices and potential output measures. Moving from the CBO's measure of potential output to a quadratic trend of output and trend unemployment rate reduces the estimates of both ϕ and β. In fact, β is statistically insignificant for some definitions of potential output. When the

GDP deflator is replaced with the CPI, the estimate of ϕ changes little while the magnitude of β increases considerably.

Unlike the Judd and Rudebusch (1998) estimates, the smoothing term ρ of CGG is much larger, implying interest rate responses to the inflation rate and output gaps are smaller. It should be stressed that the slower response implied by interest-rate-smoothing type rules is an inherent characteristic of those rules and cannot be compared directly to a basic Taylor rule similar to equation (3.1).

Although there is considerable variation in the estimated results of Taylor (1999b), Judd and Rudebusch (1998), and CGG (2000), it is clear that historical estimates of monetary policy rules are able to demonstrate that there has been a shift in the emphasis of monetary policy in the U.S., and to a large extent in other countries too, over time towards inflation control. This is evident by the fact that estimated coefficients on inflation have increased during the 1980s and early 1990s. This result is quite consistent across different studies.

4. Optimal instrument rules

Thus far the discussion focuses only on single-equation estimates of Taylor-type rules. The estimated results, as shown in the previous section, are in general not robust. Small changes in the assumptions used can yield quite different policy recommendations, leading one to question the appropriateness of using these estimated rules as policy guides. Nonetheless, Taylor-type rules are convenient tools to explain crucial aspects of monetary policy. More important, since most macroeconomic models require the specification of a policy rule, Taylor-type rules are useful as simple representations of responses by monetary authorities to changes in economic conditions. For this purpose, the choice of the relative weights on the inflation gap and the output gap in Taylor-type rules depend less on estimated results but more on how the rule interacts with the rest of the model.

In addition to the structure of the model, other criteria that determine the relative weights are the preferences of policy makers (Ball (1997), Taylor (1999a), Muscatelli *et al.* (1999), and CGG (1998)). These preferences are typically represented by a loss function defined over the variance of inflation from its target level, variation of the output gap, and frequently, the variance of the instrument itself. Given the policy makers' preferences and a particular form of policy rule, the relative weights that minimize the loss function are thus the preferred weights.

The search for the relative weights for model-based Taylor-type rules typically proceeds by stochastically simulating macro economic models with different monetary policy rules with the intent to minimize pre-specified monetary policy makers' loss functions. Policy rules that generate large variations in inflation and output are thus severely penalized. It is also a common practice to use the loss function to generate policy frontiers that measure the best obtainable pairs of unconditional variances of the output gap and the inflation rate. The points on these frontiers correspond to different preferences on the variables in the loss function. Every instrument rule on the policy frontier is efficient in the sense that it minimizes the loss function, and hence the variance of inflation (or its deviation from target), the output gap, and at times, the variance of the instrument itself. In some cases instrument variability is not included in the loss function explicitly, instead its variation is represented in the frontiers by restricting the variance of the instrument to equal some particular value. Since the rules created these ways are model-consistent, they are often termed optimal rules in the literature.

The robustness of specific Taylor-type rules across different models has received considerable attention in the literature. Levin *et al.* (1999) and Taylor (1999a) argue that Taylor-type rules are robust to model uncertainty. However, their conclusions have been questioned on the grounds that the models they use are too similar to each other. The Bank of Canada has recently completed an extensive study (Côté *et al.* (2002a, 2002b)) where they compare various Taylor-type rules across a diverse set of models of the Canadian economy. Contrary to the results of Levin *et al.* (1999) and Taylor (1999a),

they do not conclude that Taylor-type rules are robust to model uncertainty. Their study highlights the importance of model specification when choosing an interest rate rule.

Issues such as interest rate smoothing and forward-versus-backward looking time horizons that are important to estimating historical Taylor-type rules are also pertinent when estimating optimal rules. Many of the concerns raised in the earlier section resurface as they become transposed into macro economic models. The rest of the section will examine several approaches in the literature to generate optimal policy rules.

4.1 Issues related to the choice of optimal rules

One of the most important aspects when choosing an optimal rule is the channel of the monetary transmission mechanism specified in a model. Factors such as the persistence of prices and output, the interest rate elasticity of demand, the degree of openness, and the form of expectations influence the optimal parameters in an instrument rule. Therefore, identifying the channels of the monetary policy transmission process is an important step in designing an efficient instrument rule. All the models examined here follow the financial market price view of the monetary transmission mechanism. In this view monetary policy takes effect through prices and rates of return on financial assets. Some of these models are small three equation systems consisting of an aggregate demand equation, a price adjustment equation, and the instrument rule. Other models are larger ones that provide a more detailed description of aggregate demand and the price adjustment process.

The role of aggregate demand in the transmission mechanism differs between these models in three fundamental ways. First, the response of aggregate demand to changes in interest rates differs across different models. Some models assume aggregate demand responds to short-term interest rates while others assume long-term interest rates drive aggregate demand. Second, the persistence of output varies between models. This partially affects the magnitude of interest rate movements that are required to achieve output stability. Third, the inclusion of open-economy components of aggregate demand introduces an additional channel that can alter significantly the timeliness of monetary

policy. Many macroeconomic models, particularly those for the U.S., do not include an external component. As a result, optimal interest rate rules for U.S. models may not be transferable to models of open economies such as Canada.

Different models can have very different price specifications, and this determines how monetary policy affects real variables in the model. Some models assume completely backward-looking price setting behaviour, others employ more descriptive price adjustment mechanisms. Among these models staggered price or wage setting behaviour is a common assumption, in which case inflation can be entirely forward looking or be a mixture of expected future and past inflation.

Models that explicitly assume rational expectations introduce additional channels through which monetary policy can operate. Inflation expectations, and subsequent price movements they induce, enter the monetary transmission mechanism via their resulting impact on short-term real interest rates. This, combined with an expectations theory of the term structure, will in turn affect current long-term interest rate movements and subsequently aggregate demand. However, the extent to which these expectations are able to affect real interest rates also depends on the wage and price setting behaviour assumed in the model (Taylor (1995), Svensson (2000)). When the central bank has a credible inflation target, expected inflation will be quite low, reducing the need for a large weight on inflation in the instrument rule (Smets (1998)).

4.2 A general IS/Phillips-curve model and the optimal instrument rule

The section introduces a simple backward-looking structural model based on Ball (1997) to establish the linkages between the monetary transmission mechanism and the choice of an optimal instrument rule.

The model describes the economy with two equations, a dynamic IS equation (4.1) and a Phillips curve (4.2).

$$\tilde{y}_t = -\omega \tilde{r}_{t-1} + \phi \tilde{y}_{t-1} + \varepsilon_t \quad ; \qquad \omega > 0, \ 0 \le \phi \le 1 \tag{4.1}$$

$$\tilde{\pi}_t = \psi \tilde{\pi}_{t-1} + \varphi \tilde{y}_{t-1} + \eta_t \quad ; \qquad \psi > 0, \ \varphi > 0 \tag{4.2}$$

$\tilde{y}_t$ is the output gap; $\tilde{r}_t$ is the difference between the real interest rate and its equilibrium level; $\tilde{\pi}_t$ is the gap between inflation and its average level; and ε_t, η_t are white-noise shocks. Policy makers set real interest rates after observing current shocks ε_t and η_t. In this simple economic system, interest rate changes affect expected output only in the following period. Assuming one cannot forecast ε_t, expected output is given by

$$E[\tilde{y}_{t+1}] = -\omega\tilde{r}_t + \phi\tilde{y}_t \quad . \tag{4.3}$$

The policy maker can set expected output at any level by choosing the appropriate level of real interest rate. However, when doing so, the policy maker has to take expected inflation as given because the real interest rate affects inflation with a two-period lag. Thus, the policy maker has to determine expected output as a function of expected inflation:

$$\begin{aligned} E[\tilde{y}_{t+1}] &= -qE[\tilde{\pi}_{t+1}] \\ &= -qE[\psi\tilde{\pi}_t + \varphi\tilde{y}_t + \eta_{t+1}] \\ &= -q\psi\tilde{\pi}_t - q\varphi\tilde{y}_t \quad . \end{aligned} \tag{4.4}$$

From (4.3) and (4.4), the monetary policy rule can be derived as

$$\begin{aligned} \tilde{r}_t &= \left(\frac{\psi q}{\omega}\right)\tilde{\pi}_t + \left(\frac{\phi+\varphi q}{\omega}\right)\tilde{y}_t \\ &\equiv \alpha\tilde{\pi}_t + \beta\tilde{y}_t \quad . \end{aligned} \tag{4.5}$$

Both α and β are functions of the dynamics of the model. Note that equation (4.5) is identical to the original Taylor rule of equation (1.3).

The parameter q is determined by minimizing the policy makers' loss function. For a given loss function of the form

$$L = \sigma_y^2 + \mu\cdot\sigma_\pi^2 \tag{4.6}$$

where σ_y^2 is the variance of output gap, σ_π^2 is the variance of inflation gap, and μ is the weight assigned by policy makers. The solution to this minimizing problem is[8]

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[8] If ψ is set equal to 1, then the solution for q will be identical to Ball (1997). This paper uses a more general form than Ball (1997).

$$q = \frac{-(1-\psi^2+\mu\varphi^2)+\sqrt{(1-\psi^2+\mu\varphi^2)^2+4\varphi^2\psi^2\mu}}{2\psi\varphi} \quad . \tag{4.7}$$

Thus q is a function of the model's parameters and the weight that policy makers assign to inflation and output variance in their loss function. Note that the relative weights in the policy rule of (4.5) are given by

$$\alpha = \psi q / \omega \quad ; \quad \beta = \frac{\phi + \varphi q}{\omega} \quad .$$

It can be shown that q is an increasing function of μ. Therefore, as policy makers attach more weight to the variance of inflation, both α (the weight on the inflation gap) and β (the weight on the output gap) in the policy rule increase. This means that for a given shock, larger interest rate changes are needed to stabilize the economy.

Consider now the relative persistence of inflation and output in the model. If the value of ψ (the persistence of inflation) increases relative to the value of ϕ (the persistence of output), α will rise[9] relative to β. The opposite is true when ϕ increases relative to ψ. That is, the optimal parameter values in the instrument rule are positively related to the relative persistence of the corresponding variables in the model.[10] In addition, they are also affected by the response of aggregate demand to changes in the interest rate. To see this, suppose the slope of the *IS* function became steeper (that is, the size of ω increases). Clearly, α and β will fall. This results because as aggregate demand becomes more responsive to interest rates, interest rates will be not be required to move as far away from their equilibrium values to stabilize the economy.

4.3 Output-inflation variability trade-off

In this simple model, an increase in output in the previous period will increase inflation in the current period, thus inflation and output move in the same direction. In this model, increasing the coefficient on inflation or output in the policy rule will reduce the variance of both inflation and output. However, this does not hold true across all

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[9] Note that q is also a function of ψ. However, it can be shown that ψq increases with ψ.
[10] This has been observed empirically by, for example, Smets (1998), Peersman and Smets (1999) and Rudebusch (2001).

macroeconomic models. Depending on the price adjustment path assumed in the model, there can exist a trade-off in the variability between inflation and output as the relative weights on the inflation and the output gaps in the policy rule changes.

McCallum and Nelson (1999) address this issue by examining two alternative price-adjustment specifications in a small IS/LM model. Their first price-adjustment specification assumes gradual price adjustment where adjustment costs are quadratic.[11] The second price-adjustment specification assumes prices are determined one period in advance, thus movements in aggregate demand are met by output adjustments. The cost associated with altering production schedules then become the adjustment costs[12]. As a result, there exists a short-run trade-off between inflation and output variability. When prices are set in advance, an increase in aggregate demand must be met solely by an increase in output. If the interest rate response to output is large, output will return quickly to its equilibrium level; however, the adjustment costs associated with output movements will cause higher variation in inflation. Simulation results of McCallum and Nelson show that under the first price-adjustment specification, increasing the weight on either inflation or output in the policy rule while holding the other constant will reduce the variance of both inflation and output. This is not true for the second price-adjustment specification. In that case increasing the weight on output for a given weight on inflation decreases the variance of output while increasing the variance of inflation.

McCallum and Nelson (1999) highlight qualitatively the role of the price adjustment path assumed in the model. This introduces an additional element of decision making into the development of an optimal instrument rule, that is, the policy maker may have to decide which variable to stabilize at the possible expense of the other.

4.4 Response of the Taylor-type rule to demand and supply shocks

At this point it is helpful to point out that Taylor-type rules react differently to demand and supply shocks. Consider first a positive shock to aggregate demand. Output

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[11] This is the Calvo-Rotemberg model of price adjustment. See Roberts (1995) for more details about this model.

[12] This variant is referred to as the P-bar model, see McCallum (1994) for more on this model.

and prices will both increase, thus moving in the same direction. When following a Taylor-type rule the monetary authority will react by increasing interest rates, causing output to fall and prices to fall subsequently. A supply shock, on the other hand, induces a different set of responses. Take for example an increase in the price of oil. Once the oil price shock passes into the general price level and raises inflation above target, the monetary authority will react by raising interest rates. The rise in interest rates will cause output to fall below potential. This negative pressure on output subsequently causes inflation to fall back to its target level. Because inflation reacts with a lag, the shock responses of prices and output are simultaneously pushing interest rates in opposite directions. As the shock cycles through, it becomes difficult to achieve the appropriate decline in interest rates required to offset the increase in the price level while at the same time avoiding excessive downward pressure on output.

4.5 Forward-looking versus backward-looking rules

It is reasonable to assume that authorities make policy decisions based on economic conditions expected in the future. For this reason, a number of researchers prefer forward-looking, or forecast-based, policy rules to contemporaneous or backward-looking rules. Batini and Haldane (1999) argue that since lags between the implementation of monetary policy and its first effects on inflation and output are well known, one can design forecast-based rules such that they take into account these transmission lags. Failure to recognise these lags could result in cyclical instability. Moreover, expectations of monetary authorities are in general formulated based on a broad spectrum of information. In this sense, forecast-based rules are information-encompassing rules. This is not necessarily a characteristic shared by backward-looking rules.

Although there are conceptual benefits to using forecast-based rules, the choice of an optimal rule still depends on the structure of the model under consideration, particularly the specific wage-price contracting process. In cases where wage bargaining is backward looking, Batini and Haldane show that forward-looking rules serve as stabilizing mechanisms to counter-balance the backward-looking behaviour of the private

sector. On the other hand, when wages are fully flexible, there is no need for forward-looking elements in the policy rule. However, Batini and Haldane caution that in extreme cases where the monetary authority and the private sector have an excessive degree of forward-looking behaviour, forecast-based rules could be destabilizing.

Not all researchers have found clear evidence in support of forecast-based rules. Using a backward-looking model, Rudebusch and Svensson (1999) find forecast-based rules outperform the contemporaneous Taylor-type rule. The advantage, however, is only marginal. In fact, their results suggest that the contemporaneous rule is a very close second to the most favourable forecast rules. Smets (1998) uses a model based on that of Rudebusch and Svensson (1999) with the exception that potential output is endogenous. With this modification, he finds that contemporaneous rules perform similar, and marginally superior, to forecast rules. Taylor (1999a) also concludes that forecast-based rules have little advantage over contemporaneous rules after he fails to find much difference between the performance of inflation forecasts and actual inflation in his policy rule. Taylor (2000) notes that as long as forecasts are not too far out into the future, they will be very close to their contemporaneous counterparts.

4.6 Interest rate smoothing

The success of including an interest rate smoothing term in the policy rule depends largely on how the policy rule is specified and the particular model structure used. In most cases, these two factors dictate whether smoothing can speed up the stabilisation process or not.

There are a number of ways to implement interest rate smoothing. The form of the policy rule used, however, can yield quite different magnitudes of interest rate movements. For example, compare the case of adding a smoothing term to the policy rule of the simple model in equation (4.5)

$$\tilde{r}_t = \alpha\tilde{\pi}_t + \beta\tilde{y}_t + \rho\tilde{r}_{t-1} \tag{4.8}$$

to the smoothing rule of CGG (2000)

$$i_t = (1-\rho)\left[(r^* - \pi^*) - (\phi - 1)\pi^* + \phi\pi_{t,k} + \beta\tilde{y}_{t,q}\right] + \rho(L)i_{t-1} + \varepsilon_t \quad .$$

If the smoothing term ρ is close to unity, a rule like (4.8) will yield large interest rate response because it effectively adds to the previous interest rate the response to current economic conditions. On the other hand, a large smoothing term in a rule like that of CGG (2000) will yield very small changes in interest rates because they will remain close to their last period's level.

Model structure is another determining factor to the success of interest rate smoothing. Specifically, the merits of smoothing depend largely on the form of expectations assumed in the term structure of interest rates. Under the assumption of rational expectations, long-term interest rates change in accordance with expected future short-term interest rate movements. If aggregate demand is specified as a function of long-term interest rates, then policy makers can stabilize output fluctuations without the need for extensive changes in short-term interest rates (Taylor (1999a, 2000), Levin *et al.* (1999), Woodford (2001)). Levin *et al.* (1999) demonstrate this point by shortening the maturity of the relevant long rates in their models, the size of the optimal smoothing parameter declines as a result.

In the absence of rational expectations, expectations are either explicitly or implicitly backward looking. If long-term interest rates do not respond to expected future short-term interest rates, increases in long rates will be smaller and subsequently have less impact on aggregate demand. In addition, through the term structure, only a fraction of the change in short-term interest rates will be reflected in the long-term interest rate. Under these circumstances interest rate smoothing serves only to slow the response of aggregate demand. However, if aggregate demand responds to short-term interest rates instead of long-term rates, the inefficiencies caused by interest rate smoothing will be reduced.

Using the Federal Reserve Board's model[13], Williams (1999) looks explicitly at the relationship between interest rate smoothing and the form of expectations process in

[13] The important characteristic of this model is the use of rational expectations in price settings and in the term structure of interest rates.

the term structure of interest rates. He finds that first difference rules (where the parameter on the interest rate smoothing term is equal to one) perform better in terms of reducing the variance of output, inflation, and the instrument itself when expectations are rational than when VAR-based expectations are assumed. Ball (1999) provides further evidence that interest rate smoothing can lead to large inflation and output variation in a backward looking model.

A broad observation in the literature is that without rational expectations in the term structure, the feedback process takes longer. However, despite the theoretical superiority of rational expectations, the result relies heavily on how the assumption is enacted empirically. As pointed out by Taylor (1999a, 1999b), even rational expectations models are backward looking because they depend on lagged data to form future expectations. Therefore, it is not clear that properties of estimated rational expectations models (which must be based on lagged information) are significantly different from estimated backward-looking expectations models.

4.7 Exchange rate channel

For countries where foreign trade represents a significant part of their economic activity, the exchange rate can be an important channel in the monetary policy rule. Ball (1999) incorporates this exchange rate channel by modifying the IS/Phillips-curve system of equations (4.1) and (4.2) to give

$$\tilde{y}_t = -\omega' \tilde{r}_{t-1} - \sigma \tilde{e}_{t-1} + \phi' \tilde{y}_{t-1} + \varepsilon_t ; \tag{4.9}$$

$$\tilde{\pi}_t = \psi' \tilde{\pi}_{t-1} + \varphi' \tilde{y}_{t-1} - \zeta(\tilde{e}_{t-1} - \tilde{e}_{t-2}) + \eta_t ; \tag{4.10}$$

$$\tilde{e}_t = \vartheta \tilde{r}_t + \nu_t \quad . \tag{4.11}$$

All parameters are positive. Equation (4.9) is the open-economy IS curve, (4.10) is the Phillips curve, and (4.11) is the interest rate parity condition. The real exchange rate (a higher real exchange rate implies appreciation) is expressed in deviations from its mean $\tilde{e}_t = e_t - \bar{e}$. In the closed-economy model of Section 4.2, interest rate affects inflation with a two-period lag. In this modified open-economy model, interest rate affects

inflation with only a one-period lag, thus increasing the speed of monetary policy. Equations (4.9) to (4.11) can be solved together to derive a policy rule of the form

$$\gamma_1 \tilde{r}_t + (1-\gamma_1)\tilde{e}_t = \gamma_2 \tilde{y}_t + \gamma_3 (\tilde{\pi}_t + \zeta \tilde{e}_{t-1}) \quad . \tag{4.12}$$

The γs in equation (4.12) are functions of other parameters in the model. The policy variable in an open economy is now a combination of the interest rate and the exchange rate rather than just the interest rate as in the case of a closed economy. Ball (1999) interprets the left-hand side of (4.12) as the monetary condition index. The rationale being that in an open economy, policy makers have to take the interaction between the interest rate and the exchange rate into consideration when deciding on policy stances. Another departure from the closed-economy case is the use of long-term inflation (the last term in equation (4.12)) in the policy rule. This variable is designed to net out direct but temporary effects of exchange rate on inflation.

Ball calibrates this model and evaluates the policy rule of (4.12) along an output-inflation variance trade-off frontier using different combinations of parameter values. The results are then converted to a closed-economy comparable basis in order to compare the interest rate responds to changes in output and inflation. For the case where there are equal weights on inflation and output variance in the loss function, Ball finds that the open-economy optimal coefficient on the inflation gap does not change while that on the output gap is marginally larger than in the closed-economy case. Thus there is no large change in the magnitude of interest rate movements between the open and closed economy interest rate rules. It follows that for a given variance in output, an optimal open economy rule is not able to reduce the variance of inflation much further than its closed economy counterpart.

Svensson (2000) finds mixed benefits by including the exchange rate in the policy rule relative to the original Taylor rule. He develops a small open economy model similar to Ball's (1999) but using forward-looking aggregate supply and demand equations. These equations are derived under explicit microfoundations that incorporate the exchange rate channel of the monetary transmission mechanism. Overall, Svensson's simulation results suggest that the inclusion of the exchange rate in the policy rule lowers

the variance on inflation but raises the variance on output and the real interest rate relative to the Taylor rule. His results, however, can be sensitive to the preferences of the policy maker, which are reflected in the type of loss function used.

Taylor (1999) also only found weak evidence to support the inclusion of the exchange rate in a policy rule. He uses the multi-country model of Taylor (1993b) where Germany, Italy, and France are grouped into a single-currency union representing the European Monetary Union (EMU) while Britain, Japan, Canada, and the United States are assumed to conduct independent monetary policies. He simulates his model using two alterative interest rate rule: an open-economy interest rate rule of

$$\tilde{r}_t = \alpha\tilde{\pi}_t + \beta\tilde{y}_t + \chi_0\tilde{e}_t + \chi_1\tilde{e}_{t-1} \tag{4.13}$$

and a closed-economy interest rate rule of equation (4.5). The simulation results using the open-economy policy rule only show marginal improvement (in terms of reducing the variance of output and inflation) relative to those using closed-economy policy rule.

Thus far the results of Ball (1999), Svensson (2000), and Taylor (1999) provide only weak support in favour of using exchange-rate-modified Taylor rules. The results may be different for small open economies like Canada. The studies of CGG (1998, 2000) are some of those attempts to re-specify Taylor-type rules for small open economies by including foreign variables. CGG modify the reaction functions of the Bundesbank and the Bank of Japan to include the U.S. Federal funds rate and their respective real exchange rates with the U.S. In both cases these variables have small positive and significant parameters. However, for the Bundesbank's reaction function, the inclusion of the real exchange rate makes the coefficient on inflation negative.

Taylor (2001) provides an explanation for the results of Ball (1999), Svensson (2000), and Taylor (1999). He suggests that even if the parameters χ_0 and χ_1 in equation (4.13) are equal to zero in country studies of policy rules so that the interest rate does not react directly to the exchange rate, the interest rate will still react indirectly to changes in the exchange rate. The effect of movements in the exchange rate will transmit through aggregate demand and inflation channels, subsequently working their way into the

interest rate equation. Therefore, it is not vital to include the exchange rate explicitly in a policy rule. In fact, Côté *et al.* (2002a) find that open economy rules perform poorly in models of the Canadian economy. They point out that in most of the models that they consider the exchange rate acts as a shock absorber, thus any attempt to smooth fluctuations in the exchange rate impedes the adjustment process of the economy.

4.8 Measurement errors in the output gap

Similar to the results on single-equation estimation of Taylor-type rules, errors in measuring the output gap can alter the relative weights used in the optimal policy rule. Orphanides *et al.* (1999) explore the relationship between the degree of measurement error and the degree of deterioration in the performance of policy rule within the FRB/US model. Using the original Taylor rule, they show that the variance of inflation, output, and the Federal funds rate increases with the increase in the size of measurement error. Using a modified Taylor rule where some degree of interest rate smoothing is allowed, they are able to reduce the variance of inflation, output, and the interest rate relative to those using the original Taylor rule if the size of measurement error is small; however, as the size of measurement error increases, the modified Taylor rule does worse than the original Taylor rule. These results do not necessarily imply that the weight on the output gap should be set equal to zero. Orphanides *et al.* show that even in the presence of measurement error, a small weight (certainly below 0.5 of the original rule) on the output gap is still able to produce lower output and inflation variation than those of a zero weight.

Smets (1998) obtains results similar to Orphanides *et al.* (1998) for an estimated IS/Phillips-curve model of the U.S. economy. The analysis compares the weights on the output gap and the inflation gap of six alternative monetary policy rules, four of which are variants of the Taylor rule. Assuming that there is no measurement uncertainty in the output gap, Smets finds that the estimated weights on the output gap in these rules are consistently greater than the estimated weights on the inflation gap. Imposing output gap measurement uncertainty into this model alters the results markedly: the weights on the output gap in the Taylor-type rules have substantially reduced. In two of the four Taylor-

type rules, the weights on the output gap are lower than that on the inflation gap. The weights on the inflation gap, on the other hand, are only reduced marginally.[14]

4.9 Stability Condition

Taylor (1999a) uses a conventional IS curve to illustrate that an inflation parameter less than zero will generate an upward sloping demand curve and, as such, will be unstable. In this section we will show that the ability of the simple Taylor-type rule to achieve a stable equilibrium depends on the structure of the macroeconomic model in conjunction with the form of the rule.

Woodford (2002) provides a useful presentation of the stability condition for a Taylor-type rule in the context of a forward-looking IS/Phillips curve model. We use a simplified version of his model to illustrate his result. Consider the Phillips curve given by

$$\pi_t = \varphi \tilde{y}_t + \psi E_t \pi_{t+1} \tag{4.14}$$

where $\tilde{y}_t$ is the output gap, $E_t \pi_{t+1}$ is the expectation of inflation at time $t+1$ with the information available at time t. The IS equation is given by

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \omega(i_t - E_t \pi_{t+1}) \tag{4.15}$$

where i_t is the nominal interest rate. Equation (4.16) defines the Taylor-type interest rate rule

$$i = \bar{i} + \alpha(\pi_t - \bar{\pi}) + \beta(\tilde{y}_t) \tag{4.16}$$

where $\bar{i}$ is an intercept and $\bar{\pi}$ is the inflation target. Woodford applies the Blanchard and Kahn (1980) conditions for the existence of a unique solution to linear difference models to illustrate the role of α and β in achieving stability. The resulting condition is given by

$$\alpha + \frac{1-\psi}{\varphi}\beta > 1 . \tag{4.17}$$

Condition (4.17) indicates that stability depends not only on the inflation parameter α but also on the output parameter β. In this model, a large enough value of either coefficient

[14] Armour and Côté (1999) survey the literature on data and model uncertainty in further detail.

is enough to guarantee stability. CGG (1998) also employ the Blanchard and Kahn conditions to examine the stability of their Taylor-type rule in a forward looking IS/Phillips curve model. After calibrating their model, they find that as the coefficient β on the output gap increases, the lower bound on the inflation coefficient α declines. In fact, the only time α must be equal to one is when β is zero. Thus it is not necessarily the case that the interest rate must respond by more than one-for-one with inflation. It is possible to achieve stability if the interest rate also responds sufficiently to output gap changes.

Equation (4.17) also demonstrates the relationship between the slope of the Phillips curve and the values of α and β. As the Phillips curve becomes steeper (φ increases), the value of α for a given β must increase. Isard *et al.* (1999) use the Blanchard and Kahn conditions to illustrate the stability conditions of the original Taylor rule in a non-linear Phillips curve model. In their model expectations are forward looking and model consistent. They show that the original Taylor rule is able to achieve a unique equilibrium when output is at potential, but unable to reach a unique equilibrium at high levels of excess demand where the slope of the Phillips curve reaches a critical level. This result supports the results of equation (4.17) in that as φ, (the slope of the Phillips curve) gets larger, the coefficient on inflation must increase for a given coefficient on the output gap. Thus the original Taylor rule is able to achieve a unique solution at low levels of excess demand but fails to do so at high levels.

Woodford (2002) extends his analysis to include conditions for stability when the Taylor-type rule is forward looking. The forward-looking Taylor-type rule is given by equation (4.18), and the resulting stability conditions are (4.17) and (4.19). Thus when the interest rate rule is forward looking, there is both a lower (equation 4.17) and an upper (equation 4.19) bound on the value of α (for a given β) that satisfies the stability conditions.

$$i = \bar{i} + \alpha(E_t\pi_{t+1} - \bar{\pi}) + \beta(\tilde{y}_t) \tag{4.18}$$

$$\alpha < 1 + \frac{1+\psi}{\varphi}(\beta + 2\omega^{-1}) \tag{4.19}$$

Condition (4.19) implies that for values of α that are too large, adjusting interest rates in response to deviations of expected future inflation from the inflation rate target can give rise to equilibrium fluctuations due purely to self-fulfilling expectations. CGG (2000) provide evidence on the existence of an upper bound on α, although in their case the upper bound is quite high.

5. Conclusion

Taylor-type monetary policy rules stipulate that the instrument of the monetary authority (usually a short-term interest rate) reacts to two key goal variables: deviations of contemporaneous inflation from a pre-set target rate and deviations of contemporaneous real output from its potential level. The extent to which the interest rate reacts to shocks will depend on the relative weights assigned to the inflation gap and the output gap. These Taylor-type rules are simple, tractable, but at the same time capture the essence of the behaviour of the monetary authority.

Broadly speaking, there are two ways to determine the relative weights on the inflation gap and the output gap in Taylor-type rules. One is through direct estimation of a Taylor-type rule over a chosen sample period; the other is through model simulations whereby the optimal weights are those that minimize a loss function representing the preferences of policy makers defined over the trade-off between fluctuations in inflation and output. This paper reviews both approaches to designing Taylor-type rules.

It is evident in the literature that estimated Taylor-type rules are not robust. Estimated relative weights tend to shift over time (and in some cases change in sign), reflecting changes in the focus of policy stance. This means that using observations that span over long periods where switches in policy regimes are known to have taken place is not appropriate when estimating Taylor-type rules. On the other hand, using regime-

specific samples may result in small sample bias and hence the weights may not be estimated precisely.

Because specifications of Taylor-type rules are typically simple, including at times an interest rate smoothing term and an exchange rate term, empirical evidence shows that estimated relative weights are sensitive to different measurements of inflation and potential output. The problems are further complicated for countries that have no announced target rates for inflation. In those cases, additional estimation methods are needed in order to by pass these problems.

Other issues related to estimating relative weights in Taylor-type rules include the debate over the use of current versus lagged (and real-time) data, forward-looking versus backward-looking expectations, and interest rate smoothing. Thus far, empirical evidence shows that estimation results from using either current or lagged data are similar. Although policy makers are typically restricted to using real-time data when designing policies, using current-time data to estimate Taylor-type rules can be justified on the grounds that policy makers are using a broader range of information other than inflation and output. Unfortunately, there is no consensus on issues related to forward-looking versus backward-looking expectations and interest rate smoothing. Each modelling approach is able to achieve a certain degree of success in the empirical literature.

Instead of estimating the relative weights in the Taylor-type rule directly, it is a common practice to simulate a model (usually stochastically) to identify the relative weights that would minimize the value of a given loss function. However, the search for model-consistent, or optimal, Taylor-type rules depends significantly on the interaction between a specific rule and the rest of the model. Differences in dynamic structures (for example, the transmission mechanism of monetary policy) between models can lead to very different functional forms of the policy rule. In addition, the choice of relative weights is crucially determined by the preferences of policy makers, which are represented by a loss function defined over the variance of the inflation gap and the

output gap. The relative importance to which policy makers assign to fluctuations in inflation and output in the loss function will lead to different relative weights in the Taylor-type rule.

Issues such as competing definitions of inflation and potential output, forward-looking versus backward-looking expectations, current-time versus lagged data, and interest rate smoothing that are pertinent to estimated Taylor-type rules are also important considerations when designing optimal rules. Again, there is no definitive evidence to discriminate one approach in favour of the other. Much of the outcome depends crucially on the dynamic structures of the model. For example, for a given loss function, if aggregate demand in a model is sensitive to interest rate changes, then the weights in the policy rule will be lower than the case where interest elasticity is low.

An important consideration in the design of optimal Taylor-type rules for small open-economy models is whether exchange rate channels should be explicitly specified in the rule. Evidence in the literature shows that in general, Taylor-type rules that have been modified to include exchange rates explicitly do not improve the simulation performance of macroeconomic models in any significant way. Essentially the exchange rate effect is already captured in the standard form of the Taylor-rule because the effect of foreign prices is reflected in changes in aggregate demand and the cost of production.

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